

# Non-Adiabatic Holonomic Quantum Computation

Erik Sjöqvist<sup>1,2</sup>, D. M. Tong<sup>3</sup>, Björn Hessmo<sup>1</sup>, Markus Johansson<sup>2</sup>, and Kuldip Singh<sup>1</sup>

<sup>1</sup>*Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, 117543 Singapore, Singapore*

<sup>2</sup>*Department of Quantum Chemistry, Uppsala University, Box 518, Se-751 20 Uppsala, Sweden*

<sup>3</sup>*Department of Physics, Shandong University, Jinan 250100, China*

(Dated: October 20, 2011)

We develop a non-adiabatic generalization of holonomic quantum computation in which high-speed universal quantum gates can be realized by using non-Abelian geometric phases. We show how a set of non-adiabatic holonomic one- and two-qubit gates can be implemented by utilizing optical transitions in a generic three-level  $\Lambda$  configuration. Our scheme opens up for universal holonomic quantum computation on qubits characterized by short coherence times.

PACS numbers: 03.65.Vf, 03.67.Lx

Circuit-based quantum computation relies on the ability to perform a universal set of quantum gate operations on a set of quantum-mechanical bits (qubits). A key challenge to achieve this goal is to find implementations of gates that are resilient to certain kinds of errors. Holonomic quantum computation (HQC) [1] is a general procedure to build universal sets of robust gates, by using non-Abelian geometric phases.

Adiabatic holonomic gates have been proposed for trapped ions [2], superconducting nanocircuits [3], and semiconductor quantum dots [4]. These gates have turned out to be difficult to realize experimentally. An obstacle is the long run-time required for the desired parametric control associated with adiabatic evolution. In other words, as these gates operate slowly compared to the dynamical time scale, they become vulnerable to open system effects that may lead to loss of coherence. On the other hand, if the run-time is decreased in order to shorten the exposure, non-adiabatic corrections start to become significant and the parametric control is lost. These problems can be tackled by using Abelian non-adiabatic geometric phases [5] to realize quantum gates [6]. However, these gates are limited to commuting operations and thus cannot perform universal quantum computation.

To combine speed and universality, we propose in this Letter a non-adiabatic generalization of HQC. We demonstrate an experimentally feasible optical scheme to implement a universal set of holonomic one- and two-qubit gates for non-adiabatic optical transitions in three-level  $\Lambda$  configurations.

We first show how quantum holonomy arises in non-adiabatic evolution. Consider a quantum system characterized by an  $N$ -dimensional Hilbert space. Let  $C : t \mapsto M(t)$  be a smooth path of  $K$ -dimensional subspaces ( $K \leq N$ ) generated by the Hamiltonian  $H(t)$ . Mathematically,  $C$  is a path in the Grassmann manifold  $G(N; K)$ , being the set of complex  $K$ -planes in  $\mathbb{C}^N$ . If  $P(t)$  is a projector onto the subspace  $M(t)$ , then  $i\hbar\dot{P}(t) = [H(t), P(t)]$ . We consider  $t \in [0, \tau]$  such that  $P(\tau) = P(0)$ , i.e., loops in  $G(N; K)$ . Now, let  $|\zeta_k(t)\rangle$ ,  $k = 1, \dots, K$ , be a once

differentiable set of orthonormal ordered bases of  $M(t)$  along  $C$  such that  $|\zeta_k(\tau)\rangle = |\zeta_k(0)\rangle$ . The set of all bases forms a Stiefel manifold  $\mathcal{S}(N; K)$ , which is a fiber bundle with  $G(N; K)$  as base manifold and with the set of  $K \times K$  unitary matrices as fibers [7]. A lift of the loop  $C$  in  $G(N; K)$  to a loop  $\mathcal{C}$  in  $\mathcal{S}(N; K)$  corresponds to a single-valued choice of gauge. A gauge transformation is a unitary change of bases over  $C$ . The final time evolution operator projected onto the initial subspace may be written as ( $\hbar = 1$  from now on) [8]

$$U(\tau, 0) = \sum_{k,l=1}^K \left( \mathbf{T} e^{i \int_0^\tau (\mathbf{A}(t) - \mathbf{H}(t)) dt} \right)_{kl} |\zeta_k(0)\rangle \langle \zeta_l(0)|, \quad (1)$$

where  $\mathbf{T}$  is time ordering. Here,  $\mathbf{A}_{kl}(t) = i\langle \zeta_k(t) | \dot{\zeta}_l(t) \rangle$  and  $\mathbf{H}_{kl}(t) = \langle \zeta_k(t) | H(t) | \zeta_l(t) \rangle$  are Hermitian  $K \times K$  matrices.  $\mathbf{A}$  transforms as  $\mathbf{A} \rightarrow \mathbf{V}^\dagger \mathbf{A} \mathbf{V} + i\mathbf{V}^\dagger \dot{\mathbf{V}}$  under the gauge transformation  $|\zeta_k(t)\rangle \rightarrow \sum_{l=1}^K |\zeta_l(t)\rangle V_{lk}(t)$ , where  $\mathbf{V}(t)$  is a once differentiable family of unitary  $K \times K$  matrices. Thus,  $\mathbf{A}$  transforms as a proper vector potential. The unitary

$$\mathbf{U} = \mathbf{P} e^{i \oint_C \mathbf{A}} \quad (2)$$

is the holonomy matrix in the representation associated with the loop  $\mathcal{C}$ , where  $\mathbf{A}_{kl} = i\langle \zeta_k(t) | d\zeta_l(t) \rangle$  is the corresponding matrix-valued connection one-form. Note that  $\mathbf{P}$  is path ordering in  $\mathcal{S}(N; K)$  and that  $\mathbf{U} \rightarrow \mathbf{V}^\dagger(0) \mathbf{U} \mathbf{V}(0)$  under a gauge transformation. This gauge covariance essentially means that the holonomy matrix is a property of the loop  $C$  in the base manifold  $G(N; K)$ , i.e.,  $\mathbf{U} \equiv \mathbf{U}(C)$ .

Non-adiabatic HQC is realized under the following two conditions: (i) there should exist physically accessible loops  $C : [0, \tau] \ni t \mapsto M(t)$  of subspaces along which the Hamiltonian matrix  $\mathbf{H}_{kl}(t) = \langle \zeta_k(t) | H(t) | \zeta_l(t) \rangle$  vanishes for all pairs  $|\zeta_k(t)\rangle, |\zeta_l(t)\rangle \in M(t)$ ; (ii) there should exist two such loops  $C$  and  $C'$ , based on the same point in  $G(N; K)$ , for which the corresponding  $\mathbf{U}(C)$  and  $\mathbf{U}(C')$  do not commute. While the first condition assures that the evolution is purely geometric, the second one is necessary to realize universality. Under conditions (i) and

(ii), there is a set of quantum gates

$$U(\tau, 0) = U(C) = \sum_{k,l=1}^K U_{kl}(C) |\zeta_k(0)\rangle \langle \zeta_l(0)| \quad (3)$$

that may be able to perform non-adiabatically any computation on qubits encoded in  $M(0)$  based purely on the geometric properties of  $G(N; K)$ . We demonstrate that these conditions can be met in a generic three-level  $\Lambda$  configuration, by means of which a universal set of one- and two-qubit gates can be realized.

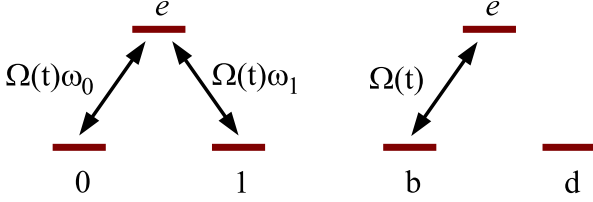


FIG. 1: (Color online) Setup for non-adiabatic holonomic one-qubit gate in a  $\Lambda$  configuration. A pair of laser pulses couple two ground state levels 0 and 1 to an excited state  $e$  (left panel). The two ground state levels define a single qubit and the laser parameters satisfy  $|\omega_0|^2 + |\omega_1|^2 = 1$ . The dark state  $|d\rangle = -\omega_1|0\rangle + \omega_0|1\rangle$  is decoupled from the bright state  $|b\rangle = \omega_0^*|0\rangle + \omega_1^*|1\rangle$  by choosing time-independent  $\omega_0$  and  $\omega_1$  over the duration of the pulse pair. The system thereby performs Rabi oscillations between the bright and excited states with frequency  $\Omega(t)$  (right panel). The evolution of the qubit subspace is purely geometric and becomes cyclic after completing a Rabi oscillation by choosing  $\Omega(t)$  to be a real-valued  $\pi$  pulse. The resulting unitary quantum gate operation acting on the qubit is determined by the holonomy of the corresponding loop in the Grassmann manifold  $G(3; 2)$ , i.e., the space of two-dimensional subspaces of the three-dimensional state space spanned by  $|0\rangle, |1\rangle$ , and  $|e\rangle$ . By applying sequentially two  $\pi$  pulse pairs with negligible temporal overlap, any desired holonomic one-qubit gate can be realized.

We first consider the one-qubit case. A pair of zero-detuned laser pulses with the same real-valued pulse envelope  $\Omega(t)$  couple selectively two ground state levels 0 and 1 to an excited state  $e$ , as shown in Fig. 1. For instance, by encoding 0, 1, and  $e$  in atomic or ionic hyperfine levels, the laser pulses can be suitably polarized so as to assure that each transition is addressed separately. The corresponding Hamiltonian reads

$$H^{(1)}(t) = \Omega(t) (\omega_0|e\rangle\langle 0| + \omega_1|e\rangle\langle 1| + \text{h.c.}). \quad (4)$$

Here, the laser parameters  $\omega_0$  and  $\omega_1$  satisfy  $|\omega_0|^2 + |\omega_1|^2 = 1$ , and describe the relative strength and relative phase of the  $0 \leftrightarrow e$  and  $1 \leftrightarrow e$  transitions. The Hamiltonian is turned on and off during the time interval  $[0, \tau]$ , controlled by  $\Omega(t)$ . We take  $|0\rangle, |1\rangle$  to define the one-qubit state space.

A universal holonomic one-qubit gate can be realized in the above  $\Lambda$  system by choosing time-independent  $\omega_0$  and  $\omega_1$  over the duration of the pulse pair. To see this, we first note that under this choice of laser parameters, the dark state  $|d\rangle = -\omega_1|0\rangle + \omega_0|1\rangle$  decouples from the dynamics, which in turn implies that the evolution is reduced to a simple Rabi oscillation between the bright state  $|b\rangle = \omega_0^*|0\rangle + \omega_1^*|1\rangle$  and the excited state [9]. The Rabi frequency is  $\Omega(t)$  and it immediately follows that the qubit subspace  $M(t)$  spanned by  $|\psi_k(t)\rangle = e^{-i \int_0^t H^{(1)}(t') dt'} |k\rangle = U(t, 0)|k\rangle$ ,  $k = 0, 1$ , undergoes cyclic evolution if the pulse-pair satisfies  $\int_0^\tau \Omega(t') dt' = \pi$ . Secondly, the evolution is purely geometric since  $\langle \psi_k(t) | H^{(1)}(t) | \psi_l(t) \rangle = \langle k | H^{(1)}(t) | l \rangle = 0$  for  $t \in [0, \tau]$ . Under the above conditions, the final time evolution operator  $U(\tau, 0)$  projected onto the computational space spanned by  $\{|0\rangle, |1\rangle\}$  defines the holonomic one-qubit gate

$$U^{(1)}(C_{\mathbf{n}}) = \mathbf{n} \cdot \boldsymbol{\sigma}, \quad (5)$$

where  $\mathbf{n}$  is a unit vector and  $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$  are the standard Pauli operators acting on  $|0\rangle, |1\rangle$ . By letting  $\omega_0 = \sin(\theta/2)e^{i\phi}$  and  $\omega_1 = -\cos(\theta/2)$ , we find  $\mathbf{n} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ .  $U^{(1)}(C_{\mathbf{n}})$  is a universal one-qubit gate. This can be seen explicitly by noting that two pairs of laser pulses corresponding to the unit vectors  $\mathbf{n}$  and  $\mathbf{m}$  applied sequentially results in

$$\begin{aligned} U^{(1)}(C) &= U^{(1)}(C_{\mathbf{m}})U^{(1)}(C_{\mathbf{n}}) \\ &= \mathbf{m} \cdot \mathbf{n} + i\boldsymbol{\sigma} \cdot (\mathbf{m} \times \mathbf{n}). \end{aligned} \quad (6)$$

This is an  $SU(2)$  transformation corresponding to a rotation of the qubit with an angle  $2 \arccos(\mathbf{n} \cdot \mathbf{m})$  around the normal of the plane in  $\mathbb{R}^3$  spanned by  $\mathbf{n}$  and  $\mathbf{m}$ . Here,  $C_{\mathbf{n}}$  and  $C_{\mathbf{m}}$  are loops in  $G(3; 2)$  based at  $M(0)$  and  $C = C_{\mathbf{m}} \circ C_{\mathbf{n}}$ . By suitable choices of  $\mathbf{n}$  and  $\mathbf{m}$ , any desired one-qubit gate can be realized. For instance, the choice  $\mathbf{n} = (\cos \phi, \sin \phi, 0)$  and  $\mathbf{m} = (\cos \phi', \sin \phi', 0)$  results in the phase shift gate  $|k\rangle \mapsto e^{2ik(\phi' - \phi)}|k\rangle$ ,  $k = 0, 1$ , up to an unimportant overall phase factor. A Hadamard gate  $|k\rangle \mapsto \frac{1}{\sqrt{2}}[(-1)^k|k\rangle + |k \oplus 1\rangle]$ ,  $k = 0, 1$ , can be implemented by a single pulse with  $\mathbf{n} = \frac{1}{\sqrt{2}}(1, 0, 1)$ .

To describe the geometric nature of the above holonomy, we lift the loop  $C_{\mathbf{n}}$  to a loop  $\tilde{C}_{\mathbf{n}}$  in  $\mathcal{S}(3; 2)$ . As noted above, each such lift corresponds to a choice of gauge. The loop  $\tilde{C}_{\mathbf{n}}$  in  $G(3; 2)$  may be represented by a set of complex 2-planes, each of which spanned by the single-valued vectors

$$\begin{aligned} |\zeta_1(t)\rangle &= U(t, 0)|d\rangle = |d\rangle, \\ |\zeta_2(t)\rangle &= e^{i\delta(t)}U(t, 0)|b\rangle \\ &= e^{i\delta(t)}[\cos \delta(t)|b\rangle - i \sin \delta(t)|e\rangle] \end{aligned} \quad (7)$$

in the three-dimensional complex vector space  $\mathbb{C}^3$ . Here,  $\delta(t) = \int_0^t \Omega(t') dt'$  and the global phase factor  $e^{i\delta(t)}$  has been inserted to ensure that  $|\zeta_2(\tau)\rangle = |\zeta_2(0)\rangle$ . The loop

$C_{\mathbf{n}}$  can be visualized by noting that  $|\zeta_1\rangle$  points in a fixed direction in  $\mathbb{C}^3$  around which  $|\zeta_2\rangle$  rotates. Physically,  $|\zeta_1\rangle$  represents the dark state and  $|\zeta_2\rangle$  describes Rabi oscillations between the bright and excited states [9]. The oscillations correspond to a loop  $C_{\mathbf{n}}$  in  $\mathcal{S}(3;2)$  represented by the single-valued gauge choice in Eq. (7) that projects onto the loop  $C_{\mathbf{n}}$  of complex 2-planes in  $G(3;2)$ . The connection one-form associated with this gauge reads

$$\mathcal{A} = \begin{pmatrix} 0 & 0 \\ 0 & \Omega(t)dt \end{pmatrix}, \quad (8)$$

which results in the holonomy matrix

$$U(C_{\mathbf{n}}) = \mathbf{Z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (9)$$

for a single Rabi oscillation. An explicit calculation confirms that  $\sum_{k,l} \mathbf{Z}_{kl} |\zeta_k(0)\rangle \langle \zeta_l(0)| = \mathbf{n} \cdot \boldsymbol{\sigma} = U^{(1)}(C_{\mathbf{n}})$ . Similarly, for the composite path  $C = C_{\mathbf{m}} \circ C_{\mathbf{n}}$ , we obtain  $\sum_{k,l} \mathbf{U}_{kl}(C) |\zeta_k(0)\rangle \langle \zeta_l(0)| = \mathbf{m} \cdot \mathbf{n} + i\boldsymbol{\sigma} \cdot (\mathbf{m} \times \mathbf{n}) = U^{(1)}(C)$  from the holonomy matrix

$$U(C) = \mathbf{W}^\dagger \mathbf{Z} \mathbf{W} \mathbf{Z}. \quad (10)$$

Here, the unitary overlap matrix with components  $\mathbf{W}_{kl} = \langle \zeta'_k(0) | \zeta_l(0) \rangle$  corresponds to an integration of a pure gauge connection one-form  $i\mathbf{V}^\dagger d\mathbf{V}$  along any path  $\mathcal{D}$  in  $\mathcal{S}(3;2)$  that connects the initial bases  $\{|\zeta_1(0)\rangle, |\zeta_2(0)\rangle\}$  and  $\{|\zeta'_1(0)\rangle, |\zeta'_2(0)\rangle\}$  of  $C_{\mathbf{n}}$  and  $C_{\mathbf{m}}$ , respectively [10]. In other words, the loop  $C = C_{\mathbf{m}} \circ C_{\mathbf{n}}$  in  $G(3;2)$  is represented by the loop  $\mathcal{C} = \mathcal{D}^{-1} \circ C_{\mathbf{m}} \circ \mathcal{D} \circ C_{\mathbf{n}}$  in  $\mathcal{S}(3;2)$ , where the four path segments correspond to the four non-commuting factors in the right-hand side of Eq. (10).

To complete the universal set, we propose a physical realization of a non-adiabatic holonomic two-qubit gate in an ion trap setup. Our scheme is a non-adiabatic version of Ref. [2], which utilizes the Sørensen-Mølmer setting [11] to design a holonomic two-qubit gate. The system consists of an array of trapped ions, each of which exhibiting an internal three-level structure  $0, 1$ , and  $e$ . The transitions  $0 \leftrightarrow e$  and  $1 \leftrightarrow e$  for an ion pair in the array is addressed by lasers with detunings  $\pm\nu \pm \delta$  and  $\pm\nu \mp \delta$ , respectively, where  $\nu$  is a phonon frequency and  $\delta$  is an additional detuning. Off-resonant couplings to the singly excited states  $|e0\rangle, |0e\rangle, |e1\rangle$ , and  $|1e\rangle$  can be suppressed by choosing the Rabi frequencies  $|\Omega_0(t)|$  and  $|\Omega_1(t)|$  smaller than  $\nu$  [11]. In this way, the effective two-ion Hamiltonian in the Lamb-Dicke regime reads

$$H^{(2)} = \frac{\eta^2}{\delta} (|\Omega_0(t)|^2 \sigma_0(\phi) \otimes \sigma_0(\phi) - |\Omega_1(t)|^2 \sigma_1(-\phi) \otimes \sigma_1(-\phi)). \quad (11)$$

Here,  $\eta$  is the Lamb-Dicke parameter ( $\eta^2 \ll 1$ ) and  $\sigma_k(\phi) = e^{i\phi/4} |e\rangle \langle k| + \text{h.c.}$ ,  $k = 0, 1$ . Note that due to the non-adiabatic nature of our gate, the ancilla state  $a$  of the original adiabatic scheme in Ref. [2] is no longer needed.

The phase  $\phi$  and ratio  $|\Omega_0(t)|^2/|\Omega_1(t)|^2 = \tan(\theta/2)$  should be kept constant during each pulse pair. These requirements imply that the two-qubit Hamiltonian matrix  $\mathbf{H}_{kl,mn}^{(2)}(t)$  vanishes, which ensures that the evolution is purely geometric. Under the  $\pi$  pulse criterion  $\frac{\eta^2}{\delta} \int_0^\tau \sqrt{|\Omega_0(t)|^4 + |\Omega_1(t)|^4} dt = \pi$ , the final time evolution operator  $U(\tau, 0)$  projected onto the computational space  $|00\rangle, |01\rangle, |10\rangle, |11\rangle$  defines the two-qubit gate

$$\begin{aligned} U^{(2)}(C) = & \cos\theta |00\rangle \langle 00| + |01\rangle \langle 01| + |10\rangle \langle 10| \\ & - \cos\theta |11\rangle \langle 11| + \sin\theta e^{-i\phi} |00\rangle \langle 11| \\ & + \sin\theta e^{i\phi} |11\rangle \langle 00|. \end{aligned} \quad (12)$$

$C$  is a path in  $\mathcal{G}(3;2)$  as the effective dynamics takes place in the three dimensional subspace spanned by  $|00\rangle, |11\rangle$ , and  $|ee\rangle$  of the internal degrees of freedom of the ions. Due to its entangling nature,  $U^{(2)}(C)$  is universal when assisted by one-qubit gates [12].

In practical implementations, the gates need to be robust to the error caused by the finite life-time of the excited state  $e$ . To test this, we add decay of  $e$  to the non-adiabatic holonomic one-qubit gate. We compare the resulting fidelity with that of the corresponding adiabatic gate. We consider the phase shift gate  $|k\rangle \rightarrow e^{ik\pi/2} |k\rangle$ ,  $k = 0, 1$ , in the non-adiabatic and adiabatic scenarios. This gate can be implemented adiabatically utilizing the  $\Lambda$ -type system, but now by varying the two laser couplings independently so as to remain approximately in an instantaneous dark state in the limit of large run-time  $T$ . We assume that the excited state decays to the auxiliary ground state level  $|g\rangle$  with rate  $\gamma$ . We model the decay with the Lindblad equation

$$\dot{\varrho}_t = -i[H^{(1)}(t), \varrho_t] + 2L\varrho_t L^\dagger - L^\dagger L\varrho_t - \varrho_t L^\dagger L, \quad (13)$$

where  $\varrho_t$  is the density operator and  $L = \sqrt{\gamma} |g\rangle \langle e|$ . Furthermore,  $H^{(1)}(t) = \Omega(t) (\omega_0 |e\rangle \langle 0| + \omega_1 |e\rangle \langle 1| + \text{h.c.})$  and  $H^{(1)}(t) = \Omega(\omega_1(t/T) |e\rangle \langle 1| + \omega_a(t/T) |e\rangle \langle a| + \text{h.c.})$  is the Hamiltonian in the non-adiabatic and adiabatic settings, respectively. In the adiabatic case, note that the  $0$ -state is decoupled from the excited state and that the  $a$  state is another ancillary ground state level [2]. Two hyperbolic secant  $\pi$  pulse pairs are chosen to implement the non-adiabatic phase shift gate. Explicitly, we choose  $\Omega(t) (\omega_0, \omega_1) = \beta \text{sech}(\beta t) (-1, 1) / \sqrt{2}$  and  $\Omega(t - \Delta t) (\omega'_0, \omega'_1) = \beta \text{sech}[\beta(t - \Delta t)] (-1, e^{-i\pi/4}) / \sqrt{2}$ , where  $\beta$  is the amplitude of the pulses and  $\Delta t$  is the temporal separation of the two pulse pairs. The ideal adiabatic gate is generated in the  $T \rightarrow \infty$  limit by varying the laser couplings  $\omega_1 = \sin(\vartheta/2) e^{i\varphi}$  and  $\omega_a = -\cos(\vartheta/2)$  along the loop  $(\vartheta, \varphi) = (0, 0) \rightarrow (\frac{\pi}{2}, 0) \rightarrow (\frac{\pi}{2}, \pi) \rightarrow (0, \pi) \rightarrow (0, 0)$  at constant speed.

In Fig. 2, we show the fidelity  $\langle \xi | U^\dagger(C) \varrho_{\text{out}} U(C) | \xi \rangle$ , computed numerically for 4000 input states  $|\xi\rangle$ , uniformly distributed over the Bloch sphere. Here,  $U(C)$  is the non-adiabatic or adiabatic holonomic gate and  $\varrho_{\text{out}}$  is

the output state computed from Eq. (13). The fidelities are plotted as functions of the dimensionless quantities  $\beta/\gamma$  and  $\Omega T$  in the non-adiabatic and adiabatic cases, respectively. Note that the pulse duration in the non-adiabatic setting decreases with increasing  $\beta/\gamma$  since the pulse area is set to the fixed value  $\pi$ . Thus, by increasing

$\beta/\gamma$  we effectively speed up the gate. Furthermore, we have chosen  $\Omega/\gamma = 12.5$  and  $\gamma\Delta t = 8$ , where the latter choice guarantees that the pulse overlap is negligible for the  $\beta/\gamma$  range shown in the figure; a necessary condition to avoid any spurious dynamical contributions to the gate.

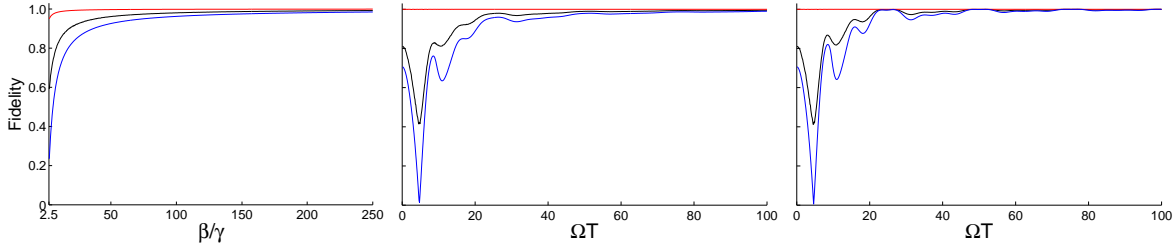


FIG. 2: Influence of decay with rate  $\gamma$  of the excited state  $e$  on the non-adiabatic and adiabatic holonomic phase shift gate  $|k\rangle \rightarrow e^{ik\pi/2}|k\rangle$ ,  $k = 0, 1$ . The effect is quantified in terms of minimum (blue), average (black), and maximum (red) fidelities. The three panels show from left to right the non-adiabatic gate with decay, the adiabatic gate with decay, and the adiabatic gate without decay. Choosing hyperbolic secant  $\pi$  pulses with amplitude  $\beta$ , the non-adiabatic fidelities are plotted as functions of the dimensionless quantity  $\beta/\gamma$ . We plot the adiabatic fidelities as functions of the dimensionless quantity  $\Omega T$ , where  $\Omega$  is time-independent global strength of laser couplings and  $T$  is the run-time of the gate. We have chosen  $\Omega/\gamma = 12.5$  and  $\gamma\Delta t = 8$ , where  $\Delta t$  is the temporal separation of the two laser pulses in the non-adiabatic setting.  $\Delta t$  is chosen sufficiently large to guarantee negligible pulse overlap for the  $\beta/\gamma$  range shown in the left panel.

The fidelities of the non-adiabatic gate tend monotonically to unity in the large  $\beta/\gamma$  limit (left panel). This demonstrates that the non-adiabatic version of the holonomic phase shift gate can be made robust to decay of the excited state by employing sufficiently short pulses. A key point with adiabatic holonomic quantum computation is that the population of the decaying excited state becomes negligible in the adiabatic limit. This behavior is confirmed as the fidelities of the adiabatic gate in the presence of decay tend to unity in the large  $T$  limit (middle panel). The oscillatory behavior, on the other hand, is due to non-adiabatic effects originating from the finite run-time of the gate and is thus present also when the decay is set to zero (right panel). The revivals seen in the fidelities of the adiabatic gate without decay have been pointed out previously in Ref. [13].

In conclusion, we have developed a non-adiabatic generalization of holonomic quantum computation (HQC) with the primary purpose to find ways to construct universal sets of robust high-speed geometric quantum gates. We have demonstrated an explicit realization of a universal set of holonomic one- and two-qubit gates in non-adiabatic evolution in three-level  $\Lambda$  configurations. The scheme requires coherent control of fewer levels and behaves simpler under decay of the excited state compared to the holonomic gates proposed for adiabatic evolution in tripod configurations [2–4]. Our gate opens up for the

possibility to perform universal quantum computation on short-lived qubits by purely geometric means.

This work was supported by the National Research Foundation and the Ministry of Education (Singapore). D.M.T. acknowledges support from the National Basic Research Program of China (Grant No. 2009CB929400).

- 
- [1] P. Zanardi and M. Rasetti, Phys. Lett. A **264**, 94 (1999).
  - [2] L.M. Duan *et al.*, Science **292**, 1695 (2001).
  - [3] L. Faoro *et al.*, Phys. Rev. Lett. **90**, 028301 (2003).
  - [4] P. Solinas *et al.*, Phys. Rev. B **67**, 121307 (2003).
  - [5] Y. Aharonov and J. Anandan, Phys. Rev. Lett. **58**, 1593 (1987).
  - [6] X. B. Wang and Matsumoto Keiji, Phys. Rev. Lett. **87**, 097901 (2001).
  - [7] K. Fujii, J. Math. Phys. **41**, 4406 (2000); I. Bengtsson and K. Życzkowski *Geometry of quantum states* (Cambridge University Press, Cambridge, 2006), Ch. 4.9.
  - [8] J. Anandan, Phys. Lett. A **133**, 171 (1988).
  - [9] M. Fleischhauer and A. S. Manka, Phys. Rev. A **54**, 794 (1996).
  - [10] D. Kult *et al.*, Phys. Rev. A **74**, 022106 (2006).
  - [11] A. Sørensen and K. Mølmer, Phys. Rev. Lett. **82**, 1971 (1999).
  - [12] M. J. Bremner *et al.*, Phys. Rev. Lett. **89**, 247902 (2002).
  - [13] G. Florio *et al.*, Phys. Rev. A **73**, 022327 (2006); A.

Trullo *et al.*, Laser Phys. **16**, 1478 (2006); C. Lupo *et al.*, Phys. Rev. A **76**, 012309 (2007).